Related Discussion
from Diff calculus

Mathematical definition of Derivative:

Let $y = f(x)$

$$y + \Delta y = f(x + \Delta x)$$

$$\cancel{y} + \Delta y - \cancel{y} = f(x + \Delta x) - f(x)$$

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Physically, very very small rate of change of ~~one~~ one variable w. r to other variable. That is rate of change of physical quantity.

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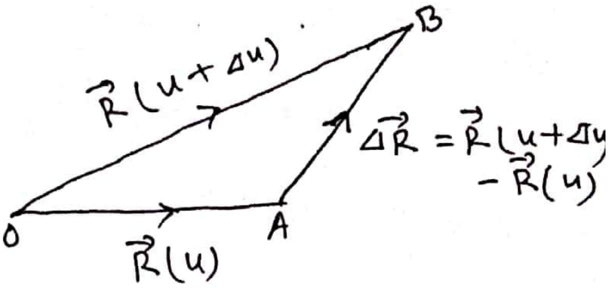
vector Differentiation

ordinary derivatives of vectors:

Let $\vec{R}(u)$ be a vector depending on a single scalar variable u, then in ΔOAB; $\Delta\vec{R} = \vec{R}(u+\Delta u) - \vec{R}(u)$



$$\frac{\Delta\vec{R}}{\Delta u} = \frac{\vec{R}(u+\Delta u) - \vec{R}(u)}{\Delta u}$$

$$\lim_{\Delta u \to 0} \frac{\Delta\vec{R}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\vec{R}(u+\Delta u) - \vec{R}(u)}{\Delta u}$$

$$\frac{d\vec{R}}{du} = \lim_{\Delta u \to 0} \frac{\vec{R}(u+\Delta u) - \vec{R}(u)}{\Delta u}, \text{ if limit exists}$$

which is derivative of vector $\vec{R}$ w.r.to u.

Q: A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time.

(i) determine its velocity and accⁿ at any time.

(ii) Find the magnitudes of velocity and accⁿ at $t=0$.

A:- (i) Let $\vec{r}$ be the position vector of the particle, then $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$$= e^{-t}\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$$

Then velocity $\vec{v} = \frac{d\vec{r}}{dt} = -e^{-t}\hat{i} - 6\sin 3t\,\hat{j} + 6\cos 3t\,\hat{k}$.

& Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = e^{-t}\hat{i} - 18\cos 3t\,\hat{j} - 18\sin 3t\,\hat{k}$.

(ii) At $t=0$; $\vec{v} = -\hat{i} - 0 + 6\hat{k} = -\hat{i} + 6\hat{k}$.

$\therefore |\vec{v}| = \sqrt{(-1)^2 + 6^2} = \sqrt{37}$.

At $t=0$; $\vec{a} = \hat{i} - 18\hat{j}$

$\therefore |\vec{a}| = \sqrt{1^2 + (-18)^2} = \sqrt{325}$

Ans:

Q. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$; where t is the time. Find the components of its velocity and acc^n at $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

A:- $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$$

velocity $\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$

at $t = 1$; $\vec{v} = 4\hat{i} - 2\hat{j} + 3\hat{k}$.

Now unit vector in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$ is

$$\hat{b} = \frac{\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{1^2 + (-3)^2 + 2^2}} = \frac{1}{\sqrt{14}}(\hat{i} - 3\hat{j} + 2\hat{k})$$

Then the component of velocity $\vec{v}$ in the given direction

$$= \vec{v} \cdot \hat{b} = (4\hat{i} - 2\hat{j} + 3\hat{k}) \cdot \frac{1}{\sqrt{14}}(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$= \frac{1}{\sqrt{14}}(4 + 6 + 6) = \frac{16}{\sqrt{14}} = \frac{8\sqrt{14}}{7}.$$

Ans:

Again, acc^n $\vec{a} = \frac{d\vec{v}}{dt} = 4\hat{i} + 2\hat{j} + 0$

$\therefore$ component of $\vec{a}$ in the given direction is

$$= \vec{a} \cdot \hat{b} = (4\hat{i} + 2\hat{j}) \cdot \frac{1}{\sqrt{14}}(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$= \frac{1}{\sqrt{14}}(4 - 6) = -\frac{2}{\sqrt{14}} = -\frac{\sqrt{14}}{7}$$